第二章　機率概論（考古題）

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2.1（95-淡江-保險）

1. Let A and B be events such that P(A) = 0.5, P(B) = 0.5, and $P(A^c \cap B^c) = \frac{1}{3}$, where $P(A^c) = 1 - P(A)$ and $P(B^c) = 1 - P(B)$. Then $P(A \cup B^c) =$

(1) $\frac{5}{6}$　(2) $\frac{4}{5}$　(3) $\frac{3}{4}$　(4) $\frac{2}{3}$　(5) $\frac{1}{2}$

【解】

$$P(A \cup B) = 1 - P(A^c \cap B^c) = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.5 + 0.5 - \frac{2}{3} = \frac{1}{3}$$

$$P(A \cup B^c) = 1 - P(A^c \cap B) = 1 - \left[P(B) - P(A \cap B)\right] = 1 - 0.5 + \frac{1}{3} = \frac{1}{2}$$

2.2（95-淡江-保險）

11. (a) A die is loaded in such a way that the probability of any particular face's showing is directly proportional to the number on that face. What is the probability that an odd number appears?

(b) A fair coin is to be tossed until a tail comes up for the first time. What is the chance of that happening on an even-numbered toss?

【解】

(a)

骰子各面出現的機率與其點數成正比，$N = 1+2+3+4+5+6 = 21$，因此

出現 n 點的機率 $= \frac{n}{21}$ ，$n = 1, 2, \ldots, 6$ 。

出現奇數點數的機率 $= \frac{1}{21} + \frac{3}{21} + \frac{5}{21} = \frac{9}{21}$ 。

(b)

第 n 次出現第一次人頭的機率 $= \left(\frac{1}{2}\right)^{n-1}\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^n$

偶次數出現人頭的機率$=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^4+\cdots=\left(\frac{1}{4}\right)^1+\left(\frac{1}{4}\right)^2+\cdots=\frac{1/4}{1-1/4}=\frac{1}{3}$

2.3（94-雲科大-資管）

1. Two balls are drawn from an urn containing 5 balls numbered from 1 to 5. The first ball is kept if it is numbered 1, and returned to the urn otherwise. What is the probability that the second ball drawn is number 2?
(A) 0.18 (B) 0.21 (C) 0.24 (D) 0.27 (E) 0.30

【解】

原則：階段內加法率，階段間乘法率。

兩階段：第一階段有抽出 1 或不是 1 兩種情形，機率分別爲$\frac{1}{5}$ 、$\frac{4}{5}$ ；

第二階段抽出 2 的機率$=\frac{1}{5}\times\frac{1}{4}+\frac{4}{5}\times\frac{1}{5}=\frac{21}{100}$ 。

2.4（94-雲科大-資管）

2. At a certain college, 20 percent of the men and 1 percent of the women are over six feet tall. Furthermore, 40 percent of the students are women. If a student is randomly picked and is observed to be over 6 feet tall, what is the probability that the student is a woman?
(A) 0.012 (B) 0.017 (C) 0.022 (D) 0.027 (E) 0.032

【解】

貝氏定理。

事前機率分組：『男、女』，事後機率分組：『矮個、高個』

計算工作表如下

事前機率 P(行$_j$|列$_i$)、P(列$_i$)

	矮個	高個	邊際機率
男	80%	20%	60%
女	99%	1%	40%

聯合機率 P(列$_i$∩行$_j$)

	矮個	高個
男	0.4800	0.1200
女	0.3960	0.0040
邊際機率	0.8760	0.1240

事後機率 P(列$_i$|行$_j$)、P(行$_j$)

	矮個	高個
男	0.5479	0.9677
女	0.4521	0.0323
邊際機率	0.8760	0.1240

因此，$P(\text{女}|\text{高個}) = 0.0323$。

2.5（94-雲科大-資管）

8. A lot of 25 color television tubes is subjected to an acceptance testing procedure. The procedure consists of drawing five tubes at random, without replacement, and testing them. If two or fewer tubes fail, the remaining ones are accepted. Otherwise the lot is rejected. Assume the lot contains four defective tubes. What is the probability of lot acceptance?
(A) 0.82 (B) 0.86 (C) 0.90 (D) 0.94 (E) 0.98

【解】

25 件中不良品有 4 件。抽檢規則：抽檢 5 件，不良品 2 件（含）以下爲通過。

若 x 爲抽中不良品件數，則其可能組合個數爲

$n(x) = C_x^4 C_{5-x}^{21}$ （4 件不良品中取 x 件，21 件正常品中取 $5-x$ 件）

全部可能組合 $N = C_5^{25}$

因此

$$P(x \le 2) = P(x=0) + P(x=1) + P(x=2)$$
$$= \frac{C_0^4 C_5^{21}}{C_5^{25}} + \frac{C_1^4 C_4^{21}}{C_5^{25}} + \frac{C_2^4 C_3^{21}}{C_5^{25}}$$
$$= \frac{20,249 + 23,940 + 7,980}{53,130} = 0.9838$$

2.6（94-雲科大-財金）

1. Bowl C contains 12 red chips and 8 blue chips. Ten of these 20 chips are selected at random and without replacement and put in bowl D, which was originally empty. Two chips are then drawn at random from bowl D. Given that these two chips are blue, find the conditional probability that 4 red chips and 6 blue chips were transferred from bowl C to bowl D.__________ (10 points)

【解】

12 紅 8 藍中抽取 10 個，其中有 4 紅 6 藍，條件爲已經抽出 2 藍。

不要用條件機率，直接將 2 藍減掉就好：

12 紅 6 藍中抽取 8 個，其中有 4 紅 4 藍，求其機率。

兩階段抽取，先決定紅，再決定藍：

$$P = \frac{C_4^{12} C_4^6}{C_{4+4}^{12+6}} = \frac{495 \times 15}{43,758} = 0.2014$$

2.7（94-逢甲-經濟）

1. A cabinet has 3 identical drawers. One drawer contains 2 gold coins, another drawer has a gold coin and a silver coin, and the third drawer has 2 silver coins. You select a drawer at random and take one of the coins, also at random.
(a) What is the chance you get a gold coin? (5%)
(b) Given that you got a gold coin, what is the chance the other coin in the drawer you selected is also gold? (5%)

【解】

(a) 完全對稱，金、銀幣沒什差別，故機率爲 $\frac{1}{2}$ 。

(b) 第一枚爲金幣，只有兩種情形，剩下來的爲 1 金 1 銀，因此金幣的機率爲 $\frac{1}{2}$ 。

2.8（94-逢甲-保險）

2. (10分) 箱子I中有6個紅球、4個白球，箱子II中有8個紅球、12個白球。若從箱子I中隨機抽取1個球後再放入箱子II中，然後再從箱子II中隨機抽取1個球。若得知第2個抽取的球是紅球，則計算第1個抽取的球也是紅球的機率。

【解】

給定第二個球的顏色，並不影響第一球的抽取機率，機率爲 $\frac{6}{6+4} = \frac{6}{10}$ 。

2.9（94-政大-風管）

1. There are 70 applicants for a job with the news department of a television station. Some of them are college graduates and some are not; some of them have at least three years' experience and some have not. The exact breakdown is as follows:

	College graduates	*Non college graduates*
At least three years' experience	14	7
Less than three years' experience	28	21

If the order in which the applicants are interviewed by the station manager is random, g is the event that the first applicant interviewed is a college graduate, and t is the event the first applicant interviewed has at least three years' experience, determine each of the following probabilities. **(15 points)**

(a) $P(g)$; (b) $P(t')$;

(c) $P(g \cap t)$; (d) $P(g' \cap t')$;

(e) $P(t|g)$; (f) $P(t'|g')$;

【解】

	g	g'	合計
t	14	7	21
t'	28	21	49
合計	42	28	70

(a) $P(g)=\frac{42}{70}=0.6$ (b) $P(t')=\frac{49}{70}=0.7$ (c) $P(g\cap t)=\frac{14}{70}=0.2$

(d) $P(g'\cap t')=\frac{21}{70}=0.3$ (e) $P(t|g)=\frac{14}{42}=0.33$ (f) $P(t'|g')=\frac{21}{28}=0.75$

2.10（94-政大-企管）

5. Sport utility vehicles (SUVs), vans, and pickups are generally considered to be more prone to roll over than cars. In a certain year, 30% of all highway fatalities involved a rollover; 18% of all fatalities in this year involved SUVs, vans, or pickups, given that the fatality involved a rollover. Given that a rollover was not involved, 10% of all fatalities involved SUVs, vans, or pickups. Consider the following definitions:

A = fatality involved an SUV, vans, or pickup

B = fatality involved a rollover

a. Use Bayes' theorem to find the probability that the fatality involved a rollover, given that the fatality involved a SUV, van, or pickup. (10 %)

b. Are SUVs, vans, or pickups generally more prone to rollover accidents? Why? (5 %)

【解】

貝氏定理。

事前機率分組：『A運動休閒車、A'』，事後機率分組：『B翻覆、B'』

計算工作表如下

事前機率 P(行$_j$|列$_i$)、P(列$_i$)

	A休閒車	非休閒車	邊際機率
B翻覆	18%	82%	30%
非翻覆	10%	90%	70%

聯合機率 P(列$_i$∩行$_j$)

	A休閒車	非休閒車
B翻覆	0.0540	0.2460
非翻覆	0.0700	0.6300
邊際機率	0.1240	0.8760

事後機率 P(列$_i$|行$_j$)、P(行$_j$)

	A休閒車	非休閒車
B翻覆	0.4355	0.2808
非翻覆	0.5645	0.7192
邊際機率	0.1240	0.8760

(a) $P\left(\text{翻覆}\middle|\text{運動休閒車}\right)=0.4355$

(b) $P\left(\text{翻覆}\middle|\text{運動休閒車}\right)=0.4355>P\left(\text{翻覆}\middle|\text{非運動休閒車}\right)=0.2808$，

運動休閒車真的比較容易翻覆。

2.11（94-台大-商學）

5. If P(A) = 0.4, P(B | A) = 0.35, P(A ∪ B) =0.69, then P(B) =

A. 0.14
B. 0.43
C. 0.75
D. 0.59

【解】

$$P\left(B\middle|A\right)=\frac{P(A\cap B)}{P(A)}=\frac{P(A\cap B)}{0.4}=0.35 \quad\Rightarrow\quad P(A\cap B)=0.14$$

$$P(A)+P(B)=P(A\cup B)+P(A\cap B)=0.69+0.14 \quad\Rightarrow\quad P(B)=0.83-0.4=0.43$$

2.12（94-台大-商學）

6. Of five letters (A, B, C, D, and E), two letters are to be selected at random. How many possible selections are there?

A. 20
B. 7
C. 5!
D. 10

【解】

$$C_2^5=\frac{5\times 4}{2\times 1}=10$$

2.13（93-雲科大-資管）

1. Roll a fair six-sided die three times. Let $A=\{1 \text{ or } 2 \text{ on the first roll}\}$, $B=\{3 \text{ or } 4 \text{ on the second roll}\}$, and $C=\{5 \text{ or } 6 \text{ on the third roll}\}$. What is the probability of $A\cup B\cup C$?

(A) 0.5 (B) 0.6329 (C) 0.6634 (D) 0.7037 (E) 0.7634

【解】

$P(A\cup B\cup C)$ 不好算，$P\left(A^c\cap B^c\cap C^c\right)$ 比較簡單，

其意義爲第一次沒擲出 1 或 2，且第二次沒擲出 3 或 4，且第三次沒擲出 5 或 6。

$$P\left(A^c\cap B^c\cap C^c\right)=\frac{4}{6}\times\frac{4}{6}\times\frac{4}{6}=\frac{8}{27}$$

故 $P(A\cup B\cup C)=1-P\left(A^c\cap B^c\cap C^c\right)=\frac{19}{27}=0.7037$

2.14（93-雲科大-資管）

2. Suppose that there are 12 songs on a compact disk (CD) of which two are your favorites. When using the random button selector on a CD player, each of the 12 selections is played once in a random order. What is the probability that the second of your two favorites (i.e., one has already been played) is the third song that is played?

(A) 0.0202 (B) 0.0303 (C) 0.0404 (D) 0.5 (E) 0.0607

【解】

兩階段，第一階段 12 挑 2 中恰有一首喜歡的，第二階段剩下 10 首選到喜歡的。

機率 $=\left(\frac{2}{12}\times\frac{10}{11}+\frac{10}{12}\times\frac{2}{11}\right)\times\frac{1}{10}=\frac{1}{33}=0.0303$

2.15（93-雲科大-資管）

3. A drawer contains four black, six brown, and eight olive socks. Two socks are selected at random from the drawer. What is the probability that both socks are olive if it is known that they are the same color?
(A) 0.3321 (B) 0.4469 (C) 0.5 (D) 0.5714 (E) 0.6438

【解】

4 黑 6 棕 8 橄欖，取 2 得相同顏色的機率分別爲

$$\frac{C_2^4}{C_2^{18}} 、 \frac{C_2^6}{C_2^{18}} 、 \frac{C_2^8}{C_2^{18}}$$

相同顏色中爲橄欖的機率

$$P(\text{橄欖}|\text{同色}) = \frac{\frac{C_2^8}{C_2^{18}}}{\frac{C_2^4}{C_2^{18}}+\frac{C_2^6}{C_2^{18}}+\frac{C_2^8}{C_2^{18}}} = \frac{C_2^8}{C_2^4+C_2^6+C_2^8} = \frac{28}{6+15+28} = \frac{28}{49} = 0.5714$$

2.16（93-雲科大-資管）

4. A package, say *A*, of 24 crocus bulbs contains 8 yellow, 8 white, and 8 purple crocus bulbs. A package, say *B*, of 24 crocus bulbs contains 6 yellow, 6 white, and 12 purple crocus bulbs. One of the two packages is selected at random. If 3 bulbs yielded 1 yellow flower, 1 white flower, and 1 purple flower, what is the conditional probability that package *A* was selected?
(A) 0.4629 (B) 0.5 (C) 0.5424 (D) 0.5838 (E) 0.6241

【解】

給定第二階段的條件不影響第一階段的抽取，因此機率爲 0.5。

2.17（93-雲科大-財金）

10. 當經濟水平爲``高''時,某一經濟指標上揚的機率是 75%; 當經濟水平爲``中''等時, 該經濟指標上揚的機率是 20%; 當經濟水平爲``低''時, 該經濟指標上揚的機率爲 5%. 在任何時期中，高經濟水平的機率和低經濟水平的機率皆爲 20%，而中等經濟水平的機率則爲 60%. 已知該指標剛剛上揚, 試問高經濟水平的機率爲何? ________________. (10 分)

【解】

貝氏定理。

事前機率分組：經濟水平『高、中、低』，事後機率分組：經濟指標『上揚，不上揚』

以下是工作表

事前機率 P(行$_j$|列$_i$)、P(列$_i$)

	上揚	不上揚	邊際機率
高	75%	25%	20%
中	20%	80%	60%
低	5%	95%	20%

聯合機率 P(列$_i$∩行$_j$)

	上揚	不上揚
高	0.1500	0.0500
中	0.1200	0.4800
低	0.0100	0.1900
邊際機率	0.2800	0.7200

事後機率 P(列$_i$|行$_j$)、P(行$_j$)

	上揚	不上揚
高	0.5357	0.0694
中	0.4286	0.6667
低	0.0357	0.2639
邊際機率	0.2800	0.7200

$P\left(\text{高}\middle|\text{上揚}\right)=0.5357$

2.18（93-雲科大-企管）

2. 兩事件 A 與 B，其中 $P(A)=0.2$ ，$P(B)=0.3$ 及 $P(A\cup B)=0.4$ 。請問：
(1) $P(A\cap B)=$ ________ 。(5%)
(2) $P(\bar{A}\cup\bar{B})=$ ________ 。(5%)
(3) $P(\bar{A}\cap\bar{B})=$ ________ 。(5%)
(4) $P(\bar{A}|B)=$ ________ 。(5%)

【解】

(1) $P(A\cap B)=P(A)+P(B)-P(A\cup B)=0.2+0.3-0.4=0.1$

(2) $P(\bar{A}\cup\bar{B})=1-P\left(\overline{\bar{A}\cup\bar{B}}\right)=1-P(A\cap B)=1-0.1=0.9$

(3) $P(\bar{A}\cap\bar{B})=1-P\left(\overline{\bar{A}\cap\bar{B}}\right)=1-P(A\cup B)=1-0.4=0.6$

(4) $P(\bar{A}|B)=1-P(\bar{\bar{A}}|B)=1-P(A|B)=1-\dfrac{P(A\cap B)}{P(B)}=1-\dfrac{0.1}{0.3}=\dfrac{2}{3}$

2.19（93-逢甲-經濟）

二.（10 分）某一社區 1,200 人口中，性別與政治傾向分配如下表：

	男性	女性	合計
民主黨	300	250	550
共和黨	350	150	500
其他	50	100	150
合計	700	500	1200

請回答下列問題：

(1)（5 分）隨機抽出一人其為女性共和黨員的機率為多少？

(2)（5 分）已知隨機抽出一女性，求其正好是共和黨員的機率為多少？

【解】

(1) $P(\text{女性} \cap \text{共和黨}) = \dfrac{150}{1200} = \dfrac{1}{8}$

(2) $P(\text{共和黨} | \text{女性}) = \dfrac{n(\text{女性} \cap \text{共和黨})}{n(\text{女性})} = \dfrac{150}{500} = \dfrac{3}{10}$

2.20（93-逢甲-經濟）

五.（10 分）一醫生研究血壓與心跳之關係。他檢查一群人記錄血壓為高、低、正常，心跳為規律、不規律，結果發現

- 14%有高血壓
- 22%有低血壓
- 15%心跳不規律
- 在心跳不規律者中有1/3高血壓
- 在正常血壓者中有1/8心跳不規律

求心跳規律且低血壓者之比例。

【解】

由給定資料將列聯表填滿。

列分組：血壓『高、低、正常』，行分組：心跳『規則、不規則』

工作表如下（已知資料）

	規則	不規則	邊際機率
高		1/3	14%
低			22%
正常		1/8	
邊際機率		15%	

工作表 II，填滿邊際機率，$64\% \times \dfrac{1}{8} = 0.08$ 、 $15\% \times \dfrac{1}{3} = 0.05$

	規則	不規則	邊際機率
高		0.05	14%
低			22%
正常		0.08	64%
邊際機率	85%	15%	

填滿後工作表

	規則	不規則	邊際機率
高	0.09	0.05	14%
低	0.20	0.02	22%
正常	0.56	0.08	64%
邊際機率	85%	15%	

$P(\text{規律}\cap\text{低})=0.2$

2.21（93-逢甲-經濟）

六. （10 分）設保險公司汽車險被保險人中有 1500 人是年輕人，8500 人是成年人。年輕人及成年人在一保單年之各申請理賠次數之機率如下：

申請理賠次數	年輕人之機率	成年人之機率
0	0.50	0.80
1	0.30	0.15
2	0.15	0.05
3	0.05	0.00

設一保單在一保單年內剛好申請理賠 1 次，求保單為年輕人保單之機率。

【解】

$$P(\text{年輕人}|\text{理賠1次})=\frac{P(\text{年輕人}\cap\text{理賠1次})}{P(\text{理賠1次})}=\frac{0.3}{0.3+0.15}=\frac{2}{3}$$

2.22（93-逢甲-工工）

1. 以下每題 6 分，共計 30 分。詳細敘述每題解題過程，否則不給分。
   (1) 已知事件 A 與 B 獨立、P(A∪B) = 0.6、P(A) = 0.3，求 P(B)。
   (2) 指數分配隨機變數具有無記憶性質，用一個實際例子說明之。
   (3) 自[0, 1]任取一數 u，說明如何用 u 產生期望值為 3 的指數分配隨機變數值。
   (4) 自[0, 2]任取兩個數，求兩個數的和大於 1 的機率。
   (5) 自 N(0, 1)取樣本大小 11 的隨機樣本，求樣本中位數為負的機率。

【解】

(1) 獨立 $\Rightarrow P(A\cap B)=P(A)P(B)$

$$P(A)+P(B)=P(A\cup B)+P(A\cap B)\quad\Rightarrow\quad 0.3+P(B)=0.6+0.3P(B)\quad\Rightarrow\quad P(B)=\frac{3}{7}$$

(2) 第三章內容。

等車時間爲指數分配，$\frac{1}{\lambda}=10$ 分鐘，若已經等 8 分鐘，則平均需再等還是 10 分鐘。

(3) 屬 OR 系統模擬內容。

(4) 第三章內容。

$$f(x,y)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$$

$$P(x+y\le 1)=\int_{x=0}^{1}\int_{y=0}^{1-x}f(x,y)\,dydx=\frac{1}{4}\int_{x=0}^{1}(1-x)\,dx=\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$$

$$P(x+y>1)=1-P(x+y\le 1)=1-\frac{1}{8}=\frac{7}{8}$$

(5) 第三章內容。

標準常態爲平均零之對稱分配，因此中位數爲正、爲負的機率相等。

計算過程如下：

二項分配：$n=11$、$p=0.5$、x：小於零次數

$$P(x\ge 6)=\left(C_6^{11}+C_7^{11}+C_8^{11}+C_9^{11}+C_{10}^{11}+C_{11}^{11}\right)\times 0.5^{11}=\frac{1024}{2048}=\frac{1}{2}$$

2.23（93-淡江-財金）

(1) 某台商於大陸上海市浦東新區投資一筆生意、發現其投資額度與其營業獲 10% 利率呈現以下聯合機率分配關係:

獲利率(Y) / 投資額(X)	10%	20%	30%
10 萬元	0.1	0.2	0.1
20 萬元	0.1	0.1	0.1
30 萬元	0.1	0.1	0.1

試求: (a) E(Y)　(b) Var(X)　(c) Cov(X,Y)　(d) E(X/y=20%)

(e) Are X & Y independent?

【解】

	10%	20%	30%	合計
10	0.1	0.2	0.1	0.4
20	0.1	0.1	0.1	0.3
30	0.1	0.1	0.1	0.3
合計	0.3	0.4	0.3	1

(a) $E(Y)=10\%\times 0.3+20\%\times 0.4+30\%\times 0.3=20\%$

$E(X)=10\times 0.4+20\times 0.3+30\times 0.3=19$

(b) $Var(X)=10^2\times0.4+20^2\times0.3+30^2\times0.3-(E(x))^2=430-19^2=69$

(c)

$$\begin{aligned}\operatorname{cov}(X,Y)&=(10\%+20\%+30\%)\times(10+20+30)\times0.1+20\%\times10\times0.1-E(X)E(Y)\\&=3.8-20\%\times19\\&=0\end{aligned}$$

(d) $E(X|y=20\%)=\dfrac{10\times0.2+20\times0.1+30\times0.1}{0.4}=17.5$

(e) X 與 Y 獨立，因 $\operatorname{cov}(X,Y)=0$

2.24（93-淡江-保險）

1. If A and B are two events, prove that $P(A\cap B)\ge 1-P(A^C)-P(B^C)$.

【解】

$$\overline{A\cap B}=\bar{A}\cup\bar{B}\quad\Rightarrow\quad P(\overline{A\cap B})=P(\bar{A}\cup\bar{B})$$

$$\Rightarrow\quad 1-P(A\cap B)=P(\bar{A})+P(\bar{B})-P(\bar{A}\cap\bar{B})$$

$$\Rightarrow\quad P(A\cap B)=1-P(\bar{A})-P(\bar{B})+P(\bar{A}\cap\bar{B})\ge1-P(\bar{A})-P(\bar{B})$$

2.25（93-淡江-保險）

2. If A, B, and C are three equally likely events and we require that $P(A\cap B\cap C)\ge 0.95$, what is P(A)?

【解】

已知$P(A)=P(B)=P(C)$，則

$$P(A)+P(B)+P(C)\ge P(A\cup B\cup C)\ge0.95\quad\Rightarrow\quad 3P(A)\ge0.95\quad\Rightarrow\quad P(A)\ge0.3167$$

2.26（93-淡江-企管）

3. 甲、乙兩人比賽擲一公正骰子，約定誰先擲出點數 6 者贏，由甲先擲，請問乙贏的機率是多少？(5%)

【解】

甲會在第 1、3、5⋯次擲時贏，總機率為各次機率加總：

$$\begin{aligned}P(\text{甲贏})&=\frac{1}{6}+\left(\frac{5}{6}\right)^2\frac{1}{6}+\left(\frac{5}{6}\right)^4\frac{1}{6}+\cdots\\&=\frac{1}{6}\left[1+\frac{25}{36}+\left(\frac{25}{36}\right)^2+\cdots\right]=\frac{1}{6}\times\frac{1}{1-25/36}=\frac{6}{11}\end{aligned}$$

2.27（93-政大-風管）

6. Three cards are randomly selected, without replacement, from an ordinary deck of 52 cards.
   a. What is the probability that all three cards are spades? (5%)
   b. What is the probability that the three cards contain one pair or three of a kind? (5%)

【解】

(a) $P\left(\text{三張黑桃}\right)=\dfrac{C_3^{13}}{C_3^{52}}=\dfrac{13\times12\times11}{52\times51\times50}=\dfrac{11}{850}$

(b) 一對包含三條，因此只要算一對的機率即可

階段 1：選一數字、再從該數字挑 2 張；階段 2：從其餘 48 張挑第三張

$$P\left(\text{一對}\right)=\dfrac{\left(C_1^{13}C_2^{4}\right)C_1^{48}}{C_3^{52}}=\dfrac{13\times6\times48}{26\times17\times50}=\dfrac{72}{425}$$

2.28（93-政大-企管）

2. A product designer wants to decide if a redundant component with a probability of 0.98 should be added in a system for a cost of $300. The system has a critical component with a probability of 0.98 of operating. System failure would involve a cost of $30,000. A $200 switch with a probability of 0.99 could be added that would automatically transfer the system to the backup component in the event of a failure. Should the backup component be added? (20%)

【解】

原系統失效的機率 $P_{old}=1-98\%=2\%$

新系統失效的機率

$$P_{new}=\text{主機器失效}\times\text{備用系統失效}=2\%\times\left(1\%\times98\%+99\%\times2\%\right)=\dfrac{5.92}{10{,}000}$$

原系統成本 $C_{old}=\$30{,}000\times2\%=\$600$

新系統成本 $C_{new}=\$300+\$200+\$30{,}000\times\dfrac{5.92}{10{,}000}=\$517.76$

因 $C_{new}<C_{old}$ ，故應加上備用系統。

2.29（93-台大-商學）

5. An analysis of personal loans at Taiwan Bank revealed the following facts: 10% of all personal loans are in default (D), 20% of those in default are homeowners (H|D), and 70% of those not in default are homeowners $(H|\overline{D})$. If one of the personal loans is selected at random, P(D|H) = ____________.
(A) 0.02 (B) 0.63 (C) 0.03 (D) 0.18 (E) None of the above.

【解】

貝氏定理。

事前機率分組：『壞帳(D)、正常』，事後機率分組：『自有住宅(H)，租屋』

以下是工作表

事前機率 P(行$_j$|列$_i$)、P(列$_i$)

	自宅(H)	租屋	邊際機率
壞帳(D)	20%	80%	10%
正常	70%	30%	90%

聯合機率 P(列$_i$∩行$_j$)

	自宅(H)	租屋
壞帳(D)	0.0200	0.0800
正常	0.6300	0.2700
邊際機率	0.6500	0.3500

事後機率 P(列$_i$|行$_j$)、P(行$_j$)

	自宅(H)	租屋
壞帳(D)	0.0308	0.2286
正常	0.9692	0.7714
邊際機率	0.6500	0.3500

$P\left(D|H\right)=0.02$

2.30（93-台大-商學）

7. The staffs of the accounting and the quality control departments rated their respective supervisor's leadership style as either (1) authoritarian or (2) participatory. Their responses are tabulated in the following table.

Department	Leadership Style		Total
	Authoritarian	Participatory	
Accounting	40	5	45
Quality Control	20	35	55
Total	60	40	100

Which of the following statements is NOT true?

(A) Accounting and Participatory are statistically independent

(B) Accounting and Quality Control are complements

(C) Accounting and Quality Control are mutually exclusive

(D) Authoritarian and participatory are collectively exhaustive

【解】

(A)false，$P(A\cap P)=\frac{5}{100}\neq P(A)P(P)=\frac{45}{100}\times\frac{40}{100}$

(B)(C)(D)true，同一行或列分組，自然互斥、周延；又只有兩組，應互爲補集。

2.31（93-台大-商學）

9. Max Sandlin is exploring the characteristics of stock market investors. He found that 60% of all investors have a net worth exceeding $1,000,000; 20% of all investors use an online brokerage; and 10% of all investors have a net worth exceeding $1,000,000 and use an online brokerage. An investor is selected randomly, and E is the event "networth exceeds $1,000,000," and O is the event "uses an online brokerage." Which of the following is true?

(A) E and O are collectively exhaustive

(B) E and O are dependent

(C) E and O are mutually exclusive

(D) E and O are independent

【解】

互斥（mutually exclusive）看$P(E\cap O)=0$

周延（collectively exhaustive）看$P(E\cup O)=1$

獨立（independent）看$P(E\cap O)=P(E)P(O)$

已知$P(E)=60\%$、$P(O)=20\%$、$P(E\cap O)=10\%$

計算$P(E\cup O)=P(E)+P(O)-P(E\cap O)=60\%+20\%-10\%=70\%$

(A)FALSE，$P(E\cup O)=70\%\neq 100\%$

(B)TRUE、(D)FALSE，$P(E\cap O)=10\%\neq P(E)P(O)=60\%\times 20\%$

(C)FALSE，$P(E\cap O)=10\%\neq 0$

2.32（93-台大-商學）

13. A CD player has a magazine that holds six CDs. The machine is capable of selecting a CD at random and then selecting a song randomly from that CD. Suppose that five CDs are albums by Paul McCartney and one is by Billy Joel. The player selects songs until a song by Billy Joel is played after which the machine is turned off. Find the probability that the machine is turned off after at least five songs have been played.
(A) 0.4823 (B) 0.729 (C) 0.935 (D) 0.231 (E) None of the above

【解】

前 4 首確定都挑到 Paul McCartney 的專輯

$P(\text{至少5首})=\left(\frac{5}{6}\right)^4=\frac{625}{1296}$，選(A)

2.33（93-台大-商學）

16. The Belgian 20-franc coin (B20), the Italian 500-lire coin (I500), and the Hong Kong 5-dollar coin (HK5) are approximately the same size. Coin purse one (C1) contains six of each of these coins. Coin purse two (C2) contains nine B20s, six I500s and three HK5s. A fair four-sided die is rolled. If the outcome is {1}, a coin is selected randomly from C1. If the outcome belongs to {2,3,4}, a coin is selected randomly from C2. Find P(C1|B20), the conditional probability that the coin was selected from C1, given that it was a Belgian coin.
(A) 2/11 (B) 2/13 (C) 4/7 (D) 3/7 (E) None of the above

【解】

貝氏定理。

事前機率分組：『C1、C2』，事後機率分組：『B20、I500、HK5』

已知 $P(C1)=\frac{1}{4},\quad P(B20|C1)=P(I500|C1)=P(HK5|C1)=\frac{6}{18}$

$$P(C1)=\frac{3}{4},\quad P(B20|C1)=\frac{9}{18},\quad P(I500|C1)=\frac{6}{18},\quad P(HK5|C1)=\frac{3}{18}$$

以下是工作表

事前機率 P(行$_j$|列$_i$)、P(列$_i$)

	B20	I500	HK5	邊際機率
C1	6/18	6/18	6/18	1/4
C2	9/18	6/18	3/18	3/4

聯合機率 P(列$_i$∩行$_j$)

	B20	I500	HK5
C1	1/12	1/12	1/12
C2	3/8	1/4	1/8
邊際機率	11/24	1/3	5/24

事後機率 P(列$_i$|行$_j$)、P(行$_j$)

	B20	I500	HK5
C1	2/11	1/4	2/5
C2	9/11	3/4	3/5
邊際機率	11/24	1/3	5/24

$$P(C1|B20)=\frac{2}{11}$$