PROVIDENCE UNIVERSITY

Midterm Spring 2005

Course: Statistics **Department: Business Administration** **Date: May 4, 2005**

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1. According to the local union president, the mean gross income of plumbers in the Salt Lake City area is normally distributed, with a mean of $45,000 and a standard deviation of $3,000. A recent investigative report for KYAK TV found, for a sample of 36 plumbers, the mean gross income was $44,000. At the .01 significant level, is it reasonable to conclude that that the mean income is not equal to $45,000? Determine the *p*-value. (10%)

(p 值法；μ=45000、雙尾、σ=3000、n=36、x=44000)

(1) 虛無假設 Ho：μ ＝ 45000（雙尾檢定）

(2) 檢定統計量 $\frac{\bar{x}-\mu}{\sigma_{\bar{x}}}$ 爲 z 分配

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3000}{\sqrt{36}} = 500$$

(3) 樣本檢定統計量值 z = (44000 – 45000) / 500 = -2

(4) z 分配，雙尾，臨界值=-2，求得機率 p = 0.0455

(5) 若決策者之顯著水準高於 0.0455 則拒絕虛無假設 Ho

2. The First National Bank of Wilson has 650 checking account customers. A recent sample of 50 of these customers showed 24 have a Visa card with the back. Construct the 95 percent confident interval for the proportion of checking account customers who have a Visa card with the bank. (10%)

(信賴區間法；p=0.5、雙尾、n=50、 $\bar{p} = 24/50 = 0.48$ 、α=0.05、N=650)

(1) 虛無假設 Ho：p ＝ 0.5（雙尾檢定）

(2) 檢定統計量 $(\bar{p}-p)/s_{\bar{p}}$ 爲 z 分配（大樣本，df=49）

$$s_{\bar{p}} = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}\sqrt{\frac{N-n}{N-1}} = 0.0679$$

(3) z 分配，信賴區間，1-α=99%，求得信賴區間

CI = { -1.96 ≦ z ≦ 1.96 } = { 0.3469 ≦ p ≦ 0.6131 }

(4) 母體參數 p = 0.5 $\in CI$

(5) 沒有充分理由來拒絕虛無假設 Ho

3. The following is sample information. Test the hypothesis at the .05 significance level that the treatment means are equal. (25%)

Treatment 1	Treatment 2	Treatment 3
9	13	10
7	16	11
11	14	15
9	13	14
8		15
10		

a. State the null hypothesis and the alternative hypothesis.
b. What is the decision rule?
c. Compute SST, SSE, and SS Total.
d. Complete an ANOVA table.
e. State your decision regarding the null hypothesis.

			ΣT =	175	
54	56	65	ΣT² =	2153	
A	B	C	A²	B²	C²
9	13	10	81	169	100
7	16	11	49	256	121
11	14	15	121	196	225
9	13	14	81	169	196
8		15	64		225
10			100		

(a)

H_0 : 三組的平均數相等 $\left(H_0 : \mu_1 = \mu_2 = \mu_3\right)$

H_1 : 三組平均數不完全相等

(b)

$F = \dfrac{MSTR}{MSE} > F_{\alpha=0.05, df=(2,12)} = 3.89$ 時拒絕虛無假設

(c)

(d)

變異來源	平方和	自由度	均方	*F 值*	*p 值*
組間變異	73.33	2	36.67	11.579	0.0016
隨機變異	38.00	12	3.17		
總和	111.33	14			

(e)

$F = \dfrac{MSTR}{MSE} = 11.58 > F_{\alpha=0.05, df=(2,12)} = 3.89 \Rightarrow$ 拒絕虛無假設，三組平均數不完全相等

4. A stockbroker at Critical Securities reported that the mean rate of return on a sample of 10 oil stocks was 12.6 percent with a standard deviation of 3.9 percent. The mean rate of return on a sample of 9 utility stocks was 10.9 percent with a standard deviation of 3.4 percent. At the .05 significant level, can we conclude that there is more variation in the oil stocks? (10%)

(z值法； $\sigma_1^2/\sigma_2^2 = 1$ 、右尾、 $n_1 = 10$ 、 $s_1 = 3.9$ 、 $n_2 = 9$ 、 $s_2 = 3.4$ 、α=0.05)

(1) 虛無假設 Ho： $\sigma_1^2/\sigma_2^2 \le 1$（右尾檢定）

(2) 檢定統計量 $\dfrac{s_1^2/s_2^2}{\sigma_1^2/\sigma_2^2}$ 爲 df=(9 , 8) 的 F 分配

(3) df=(9 , 8) 的 F 分配，右尾，α=0.05，求得拒絕區域 R = { F > 3.3881 }

(4) 樣本檢定統計量值 F = 1.3157 / 1 = 1.3157 $\notin R$

(5) 沒有充分理由來拒絕虛無假設 Ho

5. An investigation of the effectiveness of an antibacterial soap in reducing operating room contamination resulted in the accompanying table. The new soap was tested in a sample of eight operating rooms in the greater Seattle area during the last year. (10%)

	Operating Room							
	A	B	C	D	E	F	G	H
Before	6.6	6.5	9.0	10.3	11.2	8.1	6.3	11.6
After	6.8	2.4	7.4	8.5	8.1	6.1	3.4	2.0

At the 0.05 significant level, can we conclude the contamination measurements are lower after use of the new soap?

Before	6.6	6.5	9	10.3	11.2	8.1	6.3	11.6		
After	6.8	2.4	7.4	8.5	8.1	6.1	3.4	2		
d	-0.2	4.1	1.6	1.8	3.1	2	2.9	9.6	24.9	Σd
d^2	0.04	16.81	2.56	3.24	9.61	4	8.41	92.16	136.83	Σd^2

$$n = 8, \bar{d} = \frac{\Sigma d}{n} = \frac{24.9}{8} = 3.1125$$

$$s = \sqrt{\frac{\Sigma d^2 - (\Sigma d)^2 / n}{n-1}} = \sqrt{\frac{136.83 - 24.9^2/8}{7}} = 2.9113$$

(z值法；μ=0、右尾、n=8、 $\bar{d} = 3.1125$ 、s=2.9113、α=0.05)
(1) 虛無假設 Ho：d ≦ 0（右尾檢定）
(2) 檢定統計量 $(\bar{d} - d) / s_{\bar{d}}$ 爲 df=7 的 t 分配

$$s_{\bar{d}} = s / \sqrt{n} = 1.0293$$

(3) df=7 的 t 分配，右尾，α=0.05，求得拒絕區域 R = { t > 1.8946 }
(4) 樣本檢定統計量值 t = (3.1125 - 0) / 1.0293 = 3.0239 $\in R$
(5) 拒絕虛無假設 Ho

6. A recent study compared the time spent together by single- and dual-earner couples. According to the records kept by the wives during the study, the mean amount of time spent together watching television among the single-earner couples was 61 minutes per day, with a standard deviation of 15.5 minutes. For the dual-earner couples, the mean number of minutes spent watching television was 48.4 minutes, with a standard deviation of 18.1 minutes. At the .05 significance level, can we conclude that the single-earner couples on average spend more time watching television together? There were 15 single-earner and 12 dual-earner couples studied. (15%)

(z值法； $\mu_1 - \mu_2$=0、右尾、 $\sigma_1 = \sigma_2, n_1 = 15, \bar{x}_1 = 61, s_1 = 15.5, n_2 = 12, \bar{x}_2 = 48.4, s_2 = 18.1, \alpha = 0.05$)
(1) 虛無假設 Ho： $\mu_1 - \mu_2$ ≦ 0（右尾檢定）
(2) 檢定統計量 $\frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_{\bar{x}_1 - \bar{x}_2}}$ 爲 df=25 的 t 分配

$$s_{\bar{x}_1-\bar{x}_2} = s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = 6.4655$$

$$s_p = \sqrt{\frac{(15-1)(15.5)^2+(12-1)(18.1)^2}{15+12-2}} = \sqrt{278.69} = 16.694$$

(3) df=25 的 t 分配，右尾，α=0.05，求得拒絕區域 R = { t > 1.7081 }

(4) 樣本檢定統計量值 $t = \dfrac{61-48.4}{\sqrt{278.69\left(\frac{1}{15}+\frac{1}{12}\right)}} = 1.949 \in R$

(5) 拒絕虛無假設 Ho

7. If $x_1, x_2, \cdots, x_n$ are the values of a random sample from an exponential population, find the maximum likelihood estimator of its parameter θ. (15%)

Answer:

$$L(\theta) = f(x_1, x_2, \cdots, x_n; \theta)$$

$$= \prod_{i=1}^{n} f(x_i;\theta)$$

$$= \left(\frac{1}{\theta}\right)^n \cdot e^{-\frac{1}{\theta}\left(\sum_{i=1}^{n} x_i\right)}$$

$$\frac{d\left[\ln L(\theta)\right]}{d\theta} = -\frac{n}{\theta} + \frac{1}{\theta^2}\cdot\sum_{i=1}^{n} x_i = 0$$

$$\hat{\theta} = \frac{1}{n}\cdot\sum_{i=1}^{n} x_i = \bar{x}$$

Hence, the maximum likelihood estimator is $\hat{\theta} = \bar{X}$.

8. A carpet company advertises that it will deliver your carpet within 15 days of purchase. A sample of 49 past customers is taken. The average delivery time in the sample was 16.2 days with a standard deviation of 5.6 days.
a. State the null and alternative hypotheses. (5%)
b. Using a standardized test statistic, test the null hypothesis at the 5% level of significance. (5%)
c. Using a *p*-value, test the hypothesis at the 5% level of significance. (5%)
d. Compute the probability of a Type II error if the true average delivery time is 17 days after purchase. (10%)

(a),(b)
(z值法；μ=15、右尾、n=49、 $\bar{x}$ =16.2 、s=5.6、α=0.05)
(1) 虛無假設 Ho：μ ≦ 15（右尾檢定）
(2) 檢定統計量 $(\bar{x}-\mu)/s_{\bar{x}}$ 爲 z 分配（大樣本，df=48）

$$s_{\bar{x}} = s/\sqrt{n} = 0.8$$

(3) z 分配，右尾，α=0.05，求得拒絕區域 R = { z > 1.6449 }
(4) 樣本檢定統計量值 z = (16.2 - 15) / 0.8 = 1.5 $\notin R$
(5) 沒有充分理由來拒絕虛無假設 Ho

(c)
(p值法；μ=15、右尾、n=49、 $\bar{x}=16.2$ 、s=5.6)
(1) 虛無假設 Ho：μ ≦ 15（右尾檢定）
(2) 檢定統計量 $(\bar{x}-\mu)/s_{\bar{x}}$ 爲 z 分配（大樣本，df=48）

$s_{\bar{x}} = s/\sqrt{n} = 0.8$

(3) 樣本檢定統計量值 z = (16.2 - 15) / 0.8 = 1.5
(4) z 分配，右尾，臨界值=1.5，求得機率 p = 0.0668
(5) 若決策者之顯著水準高於 0.0668 則拒絕虛無假設 Ho

(d)
(求β值；μ=15、右尾、n=49、 $\bar{x}=16.2$ 、s=5.6、α=0.05、真正μ=17)
(1) 虛無假設 Ho：μ ≦ 15（右尾檢定）
(2) 檢定統計量 $(\bar{x}-\mu)/s_{\bar{x}}$ 爲 z 分配（大樣本，df=48）

$s_{\bar{x}} = s/\sqrt{n} = 0.8$

(3) z 分配，右尾，α=0.05，求得拒絕區域 R = { z > 1.6449 } = { x > 16.3159 }
(4) 臨界值之 z = (16.3159 - 17) / 0.8 = -0.8551
(5) z 分配，左尾，臨界值=-0.8551，求得機率 β = 0.1962