PROVIDENCE UNIVERSITY

Midterm Fall 2004

Course: <u>Statistics</u> **Department: <u>Business Administration</u>** **Date: <u>November 13, 2004</u>**

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Student ID: ___________________________ Name: ___________________________

Note: Please return the question sheet with your answer sheet. Fail to retun the question sheet will result in ZERO score for the exam.

1. Health issues are a concern of managers, especially as they evaluate the cost of medical insurance. A recent survey of 150 executives at Elvers Industries, a large insurance and financial firm located in the Southwest, reported the number of pounds by which the executives were overweight. Compute the mean and the standard deviation. (10%)

Pounds Overweight	Frequency
0 up to 6	14
6 up to 12	42
12 up to 18	58
18 up to 24	28
24 up to 30	8

Answer:

【習題 3.74】

f	M	$f \times M$	$M-\mu$	$(M\text{-}\mu)$^2	$f(M\text{-}\mu)$^2
14	3	42	-10.96	120.122	1681.70
42	9	378	-4.96	24.602	1033.27
58	15	870	1.04	1.082	62.73
28	21	588	7.04	49.562	1387.72
8	27	216	13.04	170.042	1360.33
		2094			5525.8

The mean is 13.96, found by 2094/150. The standard deviation is 6.07, found by the square root of 5525.8/150.

Pounds Overweight	Frequency (f)	代表數（X）	Xf	X^2	X^2f
0 up to 6	14	3	42	9	126
6 up to 12	42	9	378	81	3402
12 up to 18	58	15	870	225	13050
18 up to 24	28	21	588	441	12348
24 up to 30	8	27	216	729	5832
	150 （N）		2094 （ΣX）		34758 （ΣX^2）

$$N = 150, \Sigma X = 2094, \Sigma X^2 = 34758$$

$$\mu = \frac{\Sigma X}{N} = \frac{2094}{150} = 13.96$$

$$\sigma^2 = \frac{\Sigma X^2 - \frac{(\Sigma X)^2}{N}}{N} = \frac{34758 - \frac{(2094)^2}{150}}{150} = 36.8384$$

$$\sigma = \sqrt{36.8384} = 6.07$$

2. Recent crime report indicate that 2.5 motor vehicle thefts occur each minute in the United States. Assume that the distribution of thefts per minute can be approximated by the Poisson probability distribution.
a. Calculate the probability exactly four thefts occur in a minute. (5%)
b. What is the probability there are no thefts in two minutes? (5%)
c. What is the probability there is at least one theft in two minutes? (5%)

Answers:
【習題 6.65】

a. 0.1336, found by $\frac{(2.5)^4 e^{-2.5}}{4!}$

b. 0.0067, found by $\frac{(5)^0 e^{-5}}{0!}$

c. 0.9933, found by 1 – 0.0067

(a) $\lambda = 2.5$, 卜瓦松分配 (時間間隔一分鐘), $P(x=4) = \frac{2.5^4}{4!} e^{-2.5} = 0.1336$

(b) $\lambda = 5$, 卜瓦松分配 (時間間隔兩分鐘), $P(x=0) = e^{-5} = 0.0067$

(c) $\lambda = 5$, 卜瓦松分配 (時間間隔兩分鐘), $P(x \geq 1) = 1 - P(x=0) = 0.9933$

3. Barry Bonds of the San Francisco Giants had the highest batting average in the 2002 Major League Baseball season. His average was .370. So assume the probability of getting a hit is .370 for each time he batted. In a particular game assume he batted three times.
a. What is the probability of getting three hits in a particular game? (5%)
b. What is the probability of not getting any hits in a game? (5%)
c. What is the probability of getting at least one hit? (5%)

Answer:
【習題 5.58】

a. 0.051, found by $(0.370)^3$

b. 0.250, found by $(1 – 0.370)^3$

c. 0.750, found by (1 – 0.250)

$n = 3, p = 0.370$, 二項分配

$(a) P(x=3) = C_3^3 \times 0.370^3 \times 0.630^0 = 0.0507$

$(b) P(x=0) = C_0^3 \times 0.370^0 \times 0.630^3 = 0.2500$

$(c) P(x \geq 1) = 1 - P(x=0) = 0.7500$

4. The Ludlow Wildcats baseball team, a minor league team in the Cleveland Indians organization, plays 60 percent of their games at night and 40 percent during the day. The team wins 60 percent

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of their night games and 70 percent of their day games. According to today's newspaper, they won yesterday. What is the probability the game was played at night? (10%)

Answer:

【習題 5.35】

$$0.5625\text{, found by: } P(\text{night} \mid \text{win}) = \frac{P(\text{night}) \times P(\text{win} \mid \text{night})}{P(\text{night}) \times P(\text{win} \mid \text{night}) + P(\text{day}) \times P(\text{win} \mid \text{day})}$$

$$= \frac{0.60 \times 0.60}{[(0.60 \times 0.60)] + [(0.40 \times 0.70)]}$$

	win	loss	
day	0.4×70%		0.40
night	0.4×60%		0.60
	P(win)		

	win	loss	
day	0.28		0.40
night	0.36		0.60
	0.64		

貝氏定理：

$$P\left(night \middle| win\right) = \frac{P(night \text{ and } win)}{P(win)} = \frac{0.36}{0.64} = 0.5625$$

5. A federal study reported that 10 percent of the U.S. workforce has a drug problem. A drug enforcement official for the States of Indiana wished to investigate the statement. In his sample of 20 employed workers:

a. How many would you expected to have a drug problem? (5%)

b. What is the standard deviation? (5%)

c. What is the likelihood that none of the workers sampled has a drug problem? (5%)

d. What is the likelihood that at least one has a drug problem? (5%)

Answer:

【習題 6.49】

a. $\mu = 20(0.1) = 2$ and $\sigma = \sqrt{20(0.1)(0.9)} = 1.3416$

b. 0.121687847897, found by 1 – 0.1216

$n = 20, p = 0.1,$ 二項分配

$(a)\mu = np = 20 \times 0.1 = 2$

$(b)\sigma = \sqrt{np(1-p)} = \sqrt{20 \times 0.1 \times 0.9} = 1.3416$

$(c)P(x = 0) = C_0^{20} \times 0.1^0 \times 0.9^{20} = 0.1216$

$(d)P(x \geq 1) = 1 - P(x = 0) = 0.8784$

6. Among the 120 applicants for a job, only 80 are actually qualified. If five of the applicants are randomly selected for an in-depth interview, find the probability that only two of the five will be qualified for the job and the expected number of qualified applicants that will be selected for an in-depth interview and its standard deviation? (10%)

Answer:

n = 5, N = 120, x = 2, M = 80

$$P(x=2)=\frac{\binom{80}{2}\binom{40}{3}}{\binom{120}{5}}=0.164$$

$$E[X]=\frac{nM}{N}=\frac{5\cdot 80}{120}=3.33$$

$$Var[X]=\frac{nM(N-M)(N-n)}{N^2(N-1)}=\frac{5\cdot 80\cdot(120-80)(120-5)}{120^2(120-1)}=1.0738$$

$$\sigma=\sqrt{1.0738}=1.0362$$

$N=120, S=80, n=5,$ 超幾何分配

$$(1)P(x=2)=\frac{C_2^{80}C_3^{40}}{C_5^{120}}=\frac{80\times 79}{2}\frac{40\times 39}{2}\frac{5!}{120\times 119\times 118\times 117\times 116}=0.1638$$

$$(2)\mu=n\frac{S}{N}=5\times\frac{80}{120}=3.33$$

$$(3)\sigma=\sqrt{n\frac{S}{N}\frac{N-S}{N}\frac{N-n}{N-1}}=\sqrt{5\times\frac{80}{120}\times\frac{40}{120}\times\frac{115}{119}}=1.0362$$

7. A full house consists of three cards of one face value and 2 cards of another face value. What is the probability of getting a full house from a deck of a poker? (10%)

Answer:

Full house [3 cards of one face value and 2 cards of another face value]

Prob(Full house) = 13P2 x 4C3 x 4C2 / 52C5 = .00144058

full house $\equiv$ 三張同數字（三條）加上兩張同數字（一對）

n(全部牌組) $=C_5^{52}$

n(三條) $=C_1^{13}C_3^4$，n(一對) $=C_1^{12}C_2^4$

$$P(\text{full house})=\frac{n(\text{三條})\times n(\text{一對})}{n(\text{全部牌組})}=\frac{C_1^{13}C_3^4\times C_1^{12}C_2^4}{C_5^{52}}=\frac{13\times 4}{1}\frac{12\times 4\times 3}{2}\frac{5!}{52\times 51\times 50\times 49\times 48}=0.00144$$

8. The six numbers you pick 5 numbers out of 53 (1 to 53) and one Powerball out of 42 (1 to 42) have to exactly match the winning numbers (5 numbers + Powerball) of a lottery in order to win the

jackpot (first price). What is the expected number of times that you have to play to first win the jackpot? (10%)

Answer:

E[X]=q/p
Where p=the probability of getting a jackpot and q=1-p

$$E[X]=\frac{1-\frac{1}{\binom{53}{5}\binom{42}{1}}}{\frac{1}{\binom{53}{5}\binom{42}{1}}}$$

$n(\text{全部組合})=C_5^{53}C_1^{42}=120526770$

中樂透機率 $p=\frac{1}{n(\text{全部組合})}=\frac{1}{120526770}$

$p=\frac{1}{120526770}$,幾何分配

$\mu=\frac{1}{p}=120526770$

以上隨機變數定義爲：『作幾次』
另有隨機變數定義爲：『失敗幾次』，兩者的期望値差一次
即 $\mu=\frac{1}{p}-1=\frac{1-p}{p}=120526769$