靜宜大學企管系『統計學』小考六

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1. Colgate-Palmolive, Inc. recently developed a new toothpaste flavored with honey, They tested a group of ten people. Six of the group said they liked the new flavor, and the remaining four indicated they definitely did not. Four of the ten are selected to participate in an in-depth interview. What is the probability that of those selected for the in-depth interview two liked the new flavor and two did not?

$N=10,\ S=6,\ n=4,$ 超幾何分配

$$P(x=2)=\frac{C_2^6C_2^4}{C_4^{10}}=\frac{6\times5}{2}\times\frac{4\times3}{2}\times\frac{4\times3\times2}{10\times9\times8\times7}=\frac{3}{7}$$

2. The sales of Lexus automobiles in the Detroit area follow a Poisson distribution with a mean of 3 per day. (a) What is the probability that no Lexus is sold on a particular day? (b) What is the probability that for four consecutive days at least one Lexus is sold?

$\lambda=3,$ 卜瓦松分配

$$(a)\ P(x=0)=\frac{3^0}{0!}e^{-3}=0.0498$$

$$(b)\ P(x\ge1)=1-P(x=0)=0.9502$$

$$P(\text{連續四天至少賣一部})=\left[P(x\ge1)\right]^4=0.9502^4=0.8152$$

3. 請說明並證明卜瓦松分配（Poisson distribution）與二項分配（Binomial distribution）的關係。

若 $n\to\infty,$ 且 $np=\lambda$（常數），則二項分配成為卜瓦松分配

$$\begin{aligned}\lim_{n\to\infty}C_x^n p^x(1-p)^{n-x} &= \lim_{n\to\infty}\frac{n!}{x!(n-x)!}\left(\frac{\lambda}{n}\right)^x\left(1-\frac{\lambda}{n}\right)^{n-x}\\ &= \lim_{n\to\infty}\frac{\lambda^x}{x!}\frac{\overbrace{n\times(n-1)\times\cdots\times(n-x+1)}^{x\text{ 項}}}{n^x}\left(1-\frac{\lambda}{n}\right)^n\left(1-\frac{\lambda}{n}\right)^{-x}\\ &= \frac{\lambda^x}{x!}e^{-\lambda}\end{aligned}$$

其中

$$\lim_{n\to\infty}\frac{\overbrace{n\times(n-1)\times\cdots\times(n-x+1)}^{x\text{ 項}}}{n^x}=1,\quad \lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^n=e^{-\lambda},\quad \lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{-x}=1$$

4. 若二項分配的參數分別爲 n 與 p，請證明 $E(x)=np$ 、 $V(x)=np(1-p)$ 。

二項分配： $f(x;p,n)=C_x^n p^x (1-p)^{n-x},\ x\in\{0,1,\ldots,n\},\ 0<p<1$

$$
\begin{aligned}
E(X) &= \sum_{x=0}^{n} x C_x^n p^x (1-p)^{n-x} \\
&= np\left(p+(1-p)\right)^{n-1} \\
&= np
\end{aligned}
$$

$$
\begin{aligned}
V(X) &= E\left(X^2\right)-\left(E(X)\right)^2 \\
&= E\left(X(X-1)\right)+E(X)-\left(E(X)\right)^2 \\
&= \sum_{x=1}^{n} x(x-1)\frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} + np - (np)^2 \\
&= n(n-1)p^2 + np - (np)^2 \\
&= np(1-p)
\end{aligned}
$$

其中

$$(p+q)^n = \sum_{x=0}^{n} C_x^n p^x q^{n-x}$$

$$\sum_{x=0}^{n} x C_x^n p^x q^{n-x} = \sum_{x=0}^{n} x \frac{n}{x} p C_{x-1}^{n-1} p^{x-1} q^{n-x} = np(p+q)^{n-1}$$

$$\sum_{x=0}^{n} x(x-1) C_x^n p^x q^{n-x} = n(n-1)p^2(p+q)^{n-2}$$