行政院國家科學委員會專題研究計畫成果報告

推廣式梯形圖上支配點及相關問題研究
Domination and Related Problems on Generalized Trapezoid Graphs

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一、中文摘要

支配點 （Dominating set**）問題 與變異支配點問題可以應用在許多領域， 例如：地圖路由問題、 計算機通訊網路、 無線電廣播、 編碼問題與社群關係等等。雖然在梯形圖上解決這些支配點問題與變異支配點問題已有一些初步結果，但是這些結果中，仍然有一些部分需要研究者做進一步的改善。例如：梯形圖上的** clique domination **問題與 節點加權版本的**connected domination **問題等等。**

過去，在圖形演算法的研究中，曾經將 interval graphs **與** permutation graphs **推廣到梯形圖上，並且在許多梯形圖上的最佳化問題演算法的研究中獲得許多重要的成果。現在，研究漸漸導引到將梯形圖推廣到更高層次的方向。例如，**Flotow [6] **定義了所謂的** d-**梯形圖；**Felsner **等人** [5] **將梯形圖推廣到所謂的** (circle trapezoid graphs) **圓形梯形圖。而** Kratsch [8] **則再將** d-**梯形圖推廣到所謂的**(circular *d*-trapezoid graphs) **循環** d-**梯形圖。雖然許多最佳化問題在梯形圖上可以找到許多有效率的演算法** [2,5,9,13]**，但是在廣義梯形圖上，這幾年間卻只得到很少部分的解答。本計畫的目的就是在探討如何在一些具有特殊幾何（組合）性質的 廣義梯形圖上找到有效率的圖形演算法來解決這些變異支配點問題與其他最佳化問題。**

關鍵詞：圖形演算法、支配點問題、梯形圖、*d*-**梯形圖、圓形梯形圖、循環** *d*-**梯形圖、廣義梯形圖。。**

Abstract

Dominating set problem and its variants have many applications in areas like bus routing, communication networks, radio broadcast, code-word designs, and social networks. Many results concerning domination and its variants on trapezoid graphs have been pursposed it seems that some of these results still leave rooms for improving. Examples include the clique domination problem on the trapezoid graphs and the weighted case for connected domination problem.

Along with the direction that generalizes interval graphs and permutation graphs to (subclasses of) trapezoid graphs, researchers are now trying to generalize the class of trapezoid graphs. The class of *d-trapezoid graphs* is introduced by Flotow [6]; The *circle trapezoid graphs* is proposed by Felsner *et al.* [5]. Further, the class of *circular d-trapezoid graphs*, defined by Kratsch [8], are the intersection graphs of circular trapezoids between *d* parallel circles, also generalizing circular-arc graphs. Some of the problems may have been partly answered [5,8]; however, there may still be room for improvement. It is our purpose in this project to find more efficient algorithms on trapezoid graphs and variants of generalized trapezoid graphs.

Keywords: Graph algorithms, dominating set, Trapezoid graphs, d-trapezoid graphs, circle trapezoid graphs, circular d-trapezoid graphs, generalized trapezoid graphs..

二、緣由與目的

Many graph optimization problems, such as Hamiltonian cycle, maximum clique, maximum independent set, and colorability, have been proven to be intractable for general graphs [7]. On the other hand, there exist fast algorithms for solving these problems if the input graph has certain nice properties, or can be realized by a specific type of model.

The intersection graph of a collection of trapezoids with corner points lying on two parallel lines is called the *trapezoid graph* [3, 4]. Note that trapezoid graphs are perfect and properly contain both interval graphs and permutation graphs. Trapezoid graphs are perfect since they are cocomparability graphs.

The fastest known algorithm for recognition of trapezoid graph is given by Ma and Spinrad in [12], where they show that interval dimension 2 problem and trapezoid graphs recognition both can be solved in $O(n^2)$ time. Dagan, Golumbic, and Pinter [4] show that the channel routing problem is equivalent to the coloring problems on trapezoid graphs and present an $O(n^2)$ algorithm to solve the problem. Felsner et al. [5] design $O(n\log n)$ suring the `vulnerability' of these graphs.

In summary, circular d-trapezoid graphs generalizes d-trapezoid graphs, but circular d-trapezoid graphs do not generalize circle trapezoid graphs. Note that d-trapezoid graphs are still cocomparability graphs, but circuar d-trapezoid graphs and circle trapezoid time algorithms for chromatic number, weighted independent set, clique cover and maximum weighted clique for trapezoid graphs; the time can be improved to $O(n\log\log n)$ if the representations are sorted. It shall be noted that these results are also independently found by Chang [1]. Chen and Wang [2] show an algorithm for finding depth-first spanning trees on trapezoid graphs in $O(n)$ time. For the dominating sets problem and its variants in trapezoid graphs, see [9, 10, 13].

Along with the direction that generalizes interval graphs and permutation graphs to (subclasses of) trapezoid graphs, researchers are now trying to generalize the class of trapezoid graphs. Flotow [6] introduces the class of m-trapezoid graphs that are the intersection graphs of m-trapezoids, where an m-trapezoid is given by m+1 intervals on m+1 parallel lines. Recall that the k-th *power* of a graph $G=(V,E)$, denoted G^k, is the graph with the same vertex while two vertices are adjacent iff there exists a path of length at most k connecting them. Flotow shows that if G^k is an m-trapezoid graph then G^{k+1} is also an m-trapezoid graph. Lin [11] show that determining whether a given graph is a k-th power graph for any fixed $k>1$ is NP-complete. Felsner et al. [5] give $O(n\log^{m-1} n)$ time algorithms for chromatic number, weighted independent set, clique cover and maximum weighted clique for m-trapezoid graphs. They also propose a new class of graphs called *circle trapezoid graphs*, also known as *circular strips graphs*, that properly contains trapezoid graphs, circle graphs and circular-arc graphs as subclasses; they propose an $O(n^2)$ time algorithm for weighted independent set and an $O(n^2\log n)$ time algorithm for weighted clique problem for circle trapezoid graphs, using their algorithms for trapezoid graphs as subroutines. Note that a *circle trapezoid* is the region in a circle that lies between two non-crossing chords, and the circle trapezoid graphs are the intersection graphs of circle trapezoids in a circle. Just like *circular permutation graphs* shall not be confused with circle graphs, circle trapezoid graphs shall not be confused with *circular trapezoid graphs*, defined by Kratsch [8]. Here a *circular trapezoid* is the region in *two* circles (parallel to each other, in the 3D space) that lies between two non-crossing segments (on the cylinder surface, connecting two endpoints in each circle.) It follows that the circular trapezoid graphs are the intersection graphs of circular trapezoids between two parallel circles. They also extends circular trapezoid graphs into $d>2$ parallel circles; the generalized classes of graphs is so called circular d-trapezoid graphs. Kratsch show that polynomial time algorithms for computing the *component number vectors* and the *maximum component order vectors* for mea graphs are not subclasses of cocomparability graphs. Further, it is still not known whether we can efficiently recognize circle trapezoid graphs, $(d>2)$-trapezoid graphs, or circular $(d>= 2)$-trapezoid graphs. It seems that research has been directed towards using the the specific topological or geometric structure of theses generalized trapezoid graphs to solve more intractable optimization problems in larger classes of graphs. $(d>= 2)$-trapezoid graphs, and not much is known about problems on circle trapezoid graphs.

A dominating set of a graph $G = (V, E)$ is a subset D of V such that every vertex not in D is adjacent to at least one vertex in D. Each vertex v in V can be associated with a (non-negative) real weight, denoted by w(v). The weighted domination problem is a dominating set, D, such that its total weights w(D) is minimized. A dominating set D is independent, connected or total if the subgraph induced by D has no edge, is connected, or has no isolated vertex, respectively. Dominating set problem and its variants have many applications in areas like bus routing, communication network, radio broadcast, code-word design, and social network.

The decision version of the weighted domination problem is NP-complete even for cocomparability graphs. For trapezoid graphs, Liang shows that the minimum weighted domination and the total domination problem can be solved in O(mn) time. Lin [9] show that the minimum weighted independent domination in trapezoid graphs can be found in O(n log n) time. Srinivasan et al. [11] show that the minimum weighted connected domination problem in trapezoid graphs can be solved in O(m + n log n) time. For the unweighted case, the O(n log n) factor is improved by Liang [7], who show that the minimum-cardinality connected domination problem in trapezoid graphs can be solved in O(m + n) time. However, since the number of edges, m, can be as large as O(n^2), the potential time-complexity of the algorithm is still O(n 2).

四、計畫成果

A minimal connected dominating set is a connected dominating set such that the removal of any vertex leaves the resulting subset being no longer a connected dominating set. It is easily seen that a minimum weighted connected dominating set is minimal since the assigned weights are non-negative. It can be shown [11] that a minimal connected dominating set of a trapezoid graph is consisted of three parts: S; P, and T; here S denotes the set of a dominating source (a left dominator or a source pair), T denotes the set of a dominating target (a right dominator or a sink pair), and P denotes a (lightest) chordless path from S to T. Note that the dominating source (target) can be a singleton or a pair of vertices.

Following the previous discussion, a simple-minded algorithm can be constructed as following: for each left dominator u and source pair find those lightest paths from the dominating sources until reaching the dominating targets. Within these potentially O(n^2) lightest paths, the one with the minimum weight is the desired minimum connected dominating set. The trick to obtain an efficient algorithm is using the standard dynamic programming technique to search for the lightest paths collectively so that most of the non-optimal solutions can be overlooked. The idea here is that, after the source vertices of the trapezoid graphs being properly initialized, the aggregated weight of an undecided vertex can be calculated by other (already determined) trapezoids touching it from the left.

In our paper, *COCOON'2OOO*, (with F.R. Hsu and and Yin-Te Tsai), LNCS 1858, pp 126--136. Bondi Beach, Sydney, Australia, July 26-28, 2000, titled ``Efficient Algorithms for the Minimum Connected Domination on Trapezoid Graphs'', we show that In this paper, we show that finding the minimum cardinality connected dominating set in trapezoid graphs using *O(n)* time. For finding the minimum weighted connected dominating set, we show the problem can be efficiently solved in *O(n* loglog *n)* time.

The idea of the algorithms is by observing the minimum dominating set is decided by one of the target vertices, v, such that the corresponded aggregated weight w(v) represents the minimum weight of all connected dominating set. Clearly our algorithm works correctly if the rightmost dominating vertices of the minimum connected dominating set are actually the non-dominator sink pairs or contain one of the right dominator. It is thus not hard to verify that the calculation strategy obtains the correct answer when the size of the minimum dominating set is equal to one or greater than two.

The most complicated case of the algorithm is when the size of the connected dominating set is two. Let two adjacent vertices (u, v) form a dominating pair of the trapezoid graph. Depending on how these two trapezoids intersecting each other, we have three kinds of dominating pairs. The first case of a dominating pair is when one of the vertices is a left dominator and the other vertex is a right dominator. It is easily seen that the general scheme of the algorithm deals with the situation correctly. However, the general strategy does not necessarily compute the minimum dominating pair for the following two cases. We showed examples for such situation in the paper. The second case of a dominating pair is when one of the vertices is a dominator, but the other vertex is not a dominator. Here we show how to find the minimum weighted connected dominating pair of such kind.

Intuitively, it seems that the optimum solution will be harder to find when the size of the connected dominating is large. Surprisingly, we find that the most complicated case for both of algorithms is when the size of the connected dominating set equals to two. Especially for the weighted case, the algorithm will have to arrange several delicate settings to find the minimum weighted dominating pair, an unanticipated and interesting result before the authors investigated on the problem.

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