PROVIDENCE UNIVERSITY

Final Fall 2004

Course: **Statistics** **Department:** **Business Administration** **Date:** **January 8, 2005**

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Student ID: ________________________ Name: ________________________

Note: Please return the question sheet with your answer sheet. Fail to retun the question sheet will result in ZERO score for the exam.

1. The annual sales of romance novels follow the normal distribution. However, the mean and the standard deviation are unknown. Forty percent of the time sales are more than 470,000, and 10 percent of the time sales are more than 500,000. What are the mean and the standard deviation? (10%)

x 爲 μ,σ 的常態分配

$$\begin{cases} P(x \ge 470{,}000) = 0.4 \\ P(x \ge 500{,}000) = 0.1 \end{cases} \Rightarrow \begin{cases} z = 0.25 = \dfrac{470{,}000-\mu}{\sigma} \\ z = 1.28 = \dfrac{500{,}000-\mu}{\sigma} \end{cases} \Rightarrow \begin{cases} \mu = 462{,}718 \\ \sigma = 29{,}126 \end{cases}$$

2. A recent report in Business Week indicated that 20 percent of all employees steal from their company each year. If a company employs 50 people, what is the probability that:
a. Fewer than 5 empolyees steal? (5%)
b. More than 5 employees steal? (5%)
c. Exactly 5 employees steal? (5%)
d. More than 5 but fewer than 15 employees steal? (5%)

x 爲 $n=50, p=0.2$ 的二項分配 $\Rightarrow$ y 爲 $\mu = np = 10, \sigma = \sqrt{np(1-p)} = 2.83$ 的常態分配

$$(a) P(x<5) = P(y \le 4.5) = P(z \le -1.94) = 0.0262 \quad \left(\frac{4.5-10}{2.83} = -1.94\right)$$

$$(b) P(x>5) = P(y \ge 5.5) = P(z \ge -1.59) = 0.9441 \quad \left(\frac{5.5-10}{2.83} = -1.59\right)$$

$$(c) P(x=5) = P(4.5 \le y \le 5.5) = P(-1.94 \le z \le -1.59) = 0.0297$$

$$(d) P(5<x<15) = P(5.5 \le y \le 14.5) = P(-1.59 \le z \le 1.59) = 0.8882$$

3. A recent report in USA today indicated a typical family of four spends $490 per month on food. Assume the distribution of food expenditures for a family of four follows the normal distribution, with a mean of $490 and a standard deviation of $90.
a. What percent of the families spend more than $30 but less than $490 per month on food? (5%)
b. What percent of the families spend less than $430 per month on food? (5%)
c. What percent of the families spend between $430 and $600 per month on food? (5%)
d. What percent of the families spend between $500 and $600 per month on food? (5%)

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x 爲 $\mu = 490, \sigma = 90$ 的常態分配

$(a) P(30 \le x \le 490) = P(-5.11 \le z \le 0) = 0.5000 \quad \left(\frac{30-490}{90} = -5.11\right)$

$(b) P(x \le 430) = P(z \le -0.67) = 0.2514 \quad \left(\frac{430-490}{90} = -0.67\right)$

$(c) P(430 \le x \le 600) = P(-0.67 \le z \le 1.22) = 0.6374 \quad \left(\frac{600-490}{90} = 1.22\right)$

$(d) P(500 \le x \le 600) = P(0.11 \le z \le 1.22) = 0.3450 \quad \left(\frac{500-490}{90} = 0.11\right)$

4. The Sony Corporation produces a Walkman that requires two AA batteries. The mean life of these batteries in this product is 35. The distribution of the battery lives closely follows the normal probability distribution with a standard deviation of 5.5 hours. As a part of their testing program Sony tests samples of 25 batteries.
a. What can you say about the shape of the distribution of sample mean? (5%)
b. What is the standard error of the distribution of the sample mean? (5%)
c. What proportion of the sample will have a mean useful life of more than 36 hours? (5%)
d. What proportion of the sample will have a mean useful life greater than 34.5 hours? (5%)
e. What proportion of the sample will have a mean useful life between 34.5 and 36.0 hours? (5%)

x 爲 $\mu = 35, \sigma = 5.5$ 的常態分配 $\Rightarrow \bar{x}$ 爲 $\mu_{\bar{x}} = 35, \sigma_{\bar{x}} = 5.5/\sqrt{25} = 1.1$ 的常態分配 $\left(\bar{x} = \frac{x_1 + x_2 + \cdots + x_{25}}{25}\right)$

(a)常態的鐘形（bell shape）

$(b) \sigma_{\bar{x}} = 5.5/\sqrt{25} = 1.1$

$(c) P(\bar{x} \ge 36) = P(z \ge 0.91) = 0.1814 \quad \left(\frac{36-35}{1.1} = 0.91\right)$

$(d) P(\bar{x} \ge 34.5) = P(z \ge -0.45) = 0.6736 \quad \left(\frac{34.5-35}{1.1} = -0.45\right)$

$(e) P(34.5 \le \bar{x} \le 36) = P(-0.45 \le z \le 0.91) = 0.4922$

5. The probabilities are 0.40, 0.50, and, 0.10 that, in city driving, a certain kind of compact car will average less than 22 miles per gallon, from 22 to 26 miles per gallon, or more than 26 miles per gallon. Find the probability that among ten such cars tested, three will average less than 22 miles per gallon, six will average from 22 to 26 miles per gallon, and one will average more than 26 miles per gallon. (5%)

成功次數 (x_1, x_2, x_3) 爲 $n = 10, p_1 = 0.4, p_2 = 0.5, p_3 = 0.1$ 的多項分配（multinomial distribution），機率函數爲

$$P(x_1, x_2, x_3) = C^n_{x_1} p_1^{x_1} \times C^{n-x_1}_{x_2} p_2^{x_2} \times C^{n-x_1-x_2}_{x_3} p_3^{x_3} = \frac{n!}{x_1! x_2! x_3!} p_1^{x_1} p_2^{x_2} p_3^{x_3}$$

即 $P(3,6,1) = \frac{10!}{3! \cdot 6! \cdot 1!}(0.4)^3 (0.5)^6 (0.1)^1 = 0.042$

6. Show that $f(x)=3x^2$ for $0<x<1$ represent a density function, and calculate the probability that $0.1<x<0.5$. (10%)

$$(a)P(-\infty<x<\infty)=1 \;\Rightarrow\; \int_{-\infty}^{\infty} f(x)dx=\int_0^1 3x^2dx=\left[x^3\right]_0^1=1$$

$$(b)P(0.1\le x\le 0.5)=\int_{0.1}^{0.5}3x^2dx=\left[x^3\right]_{0.1}^{0.5}=0.124$$

7. If the joint probability density of X and Y is given by

$$f(x,y)=\begin{cases}\frac{1}{4}(2x+y) & \text{for } 0<x<1, 0<y<2\\ 0 & \text{elsewhere}\end{cases}$$

find

a. the marginal density of X. (5%)

b. the conditional density of Y given $X=\frac{1}{4}$. (5%)

$$(a)f(x)=\int_{-\infty}^{\infty} f(x,y)dy=\int_0^2\frac{1}{4}(2x+y)dy=\frac{1}{4}\left[2xy+\frac{1}{2}y^2\right]_0^2=x+\frac{1}{2},\quad 0<x<1$$

$$(b)f\left(y\middle|x=\tfrac{1}{4}\right)=\frac{f\left(x=\frac{1}{4},y\right)}{f\left(x=\frac{1}{4}\right)}=\frac{\frac{1}{4}(2\times\frac{1}{4}+y)}{\frac{1}{4}+\frac{1}{2}}=\frac{1}{6}+\frac{1}{3}y,\quad 0<y<2$$

8. The lifetime of a light bulb is exponentially distributed with a mean of 1,000 days. What is the probability that the lifetime exceeds 1,000 days? (10%)

x 爲 $\frac{1}{\lambda}=1{,}000\ \left(\lambda=\frac{1}{1{,}000}\right)$ 的指數分配

$$P(x>1{,}000)=\int_{1{,}000}^{\infty}\lambda e^{-\lambda x}dx=\int_{1{,}000}^{\infty}\frac{1}{1{,}000}e^{-\frac{1}{1{,}000}x}dx=\left[-e^{-\frac{1}{1{,}000}x}\right]_{1{,}000}^{\infty}=e^{-\frac{1}{1{,}000}\times 1{,}000}=e^{-1}=0.368$$

或 $P(x>a)=1-P(x\le a)=e^{-\lambda a}\;\Rightarrow\; P(x>1{,}000)=e^{-\frac{1}{1{,}000}\times 1{,}000}=e^{-1}=0.368$